Learning in Big Data Analytics
Support Vector Machines

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Perceptrons Revisited

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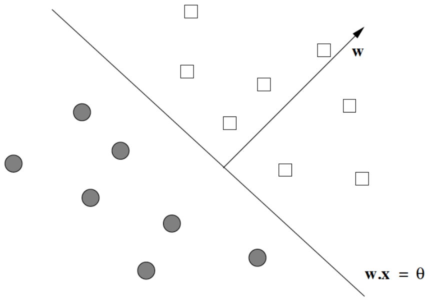

- A perceptron divides the space into two half spaces
- Half spaces capture the two different classes
- Normal vector alternative description of half space

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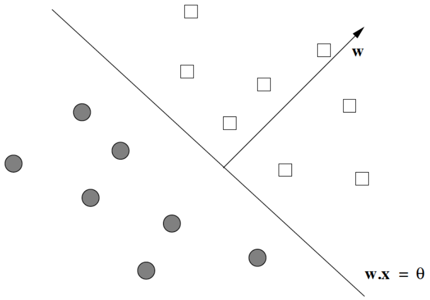

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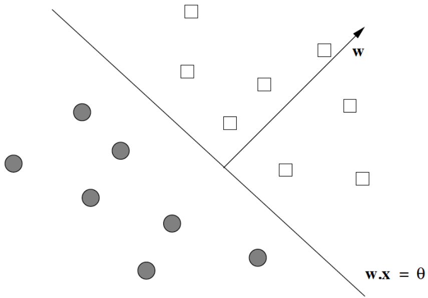

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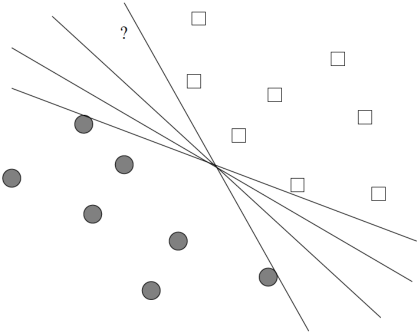

- Several half spaces (normal vectors) divide training data
- *Question:* any half space optimal, in a sensibly defined way?
- What to do if data cannot be separated (is *non-separable*)?

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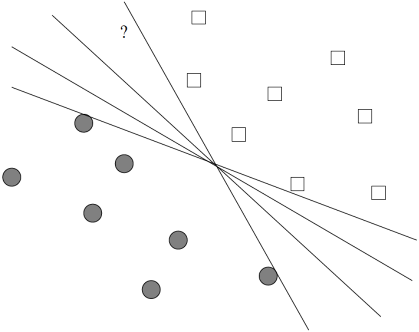

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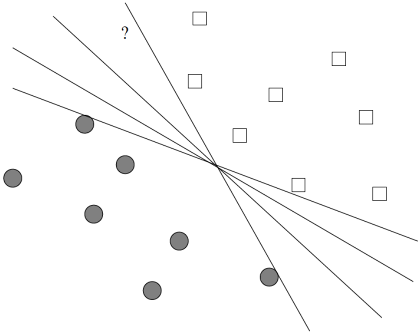

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Support Vector Machines: Motivation

- Support vector machines (SVM's) address to choose most reasonable half space
- SVM's choose half space that maximizes the *margin*, i.e. the distance between data points and half space
- If separable, maximize distance between hyperplane and closest data points
- If not separable, minimize *loss function* that
  - penalizes misclassified points
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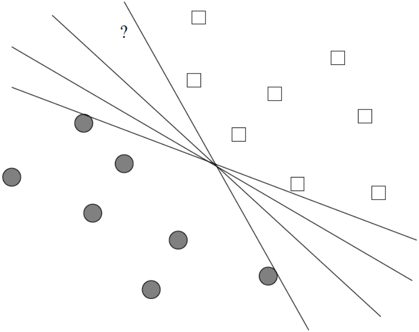
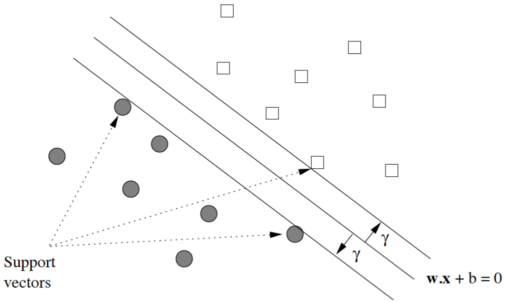


- Outer hyperplanes come very close to data points
- So, inner hyperplanes are likely the better choice
- ☞ Try to make explicit!

Separable Data

Separable Data



- *Goal:* Select hyperplane $\mathbf{w} \cdot \mathbf{x} + b = 0$ that maximizes distance γ
- *Intuition:* The further away data from hyperplane, the more certain their classification
- Increases chances to correctly classify unseen data (to generalize)

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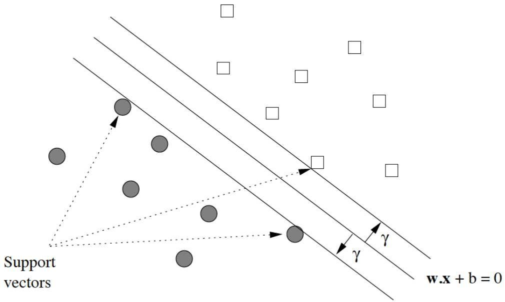

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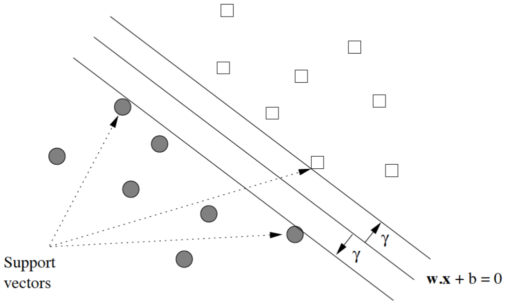
Support Vectors



- Two parallel hyperplanes at distance γ touch one or more of *support vectors*
- In most cases, d-dimensional data set has $d + 1$ support vectors (but there can be more)

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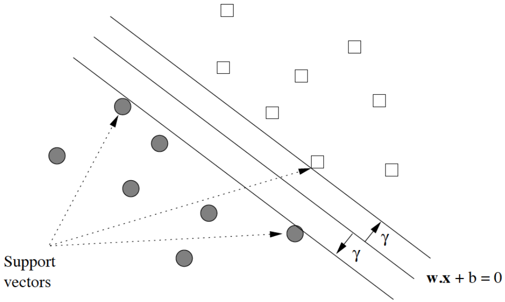

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Problem Formulation: First Try

Let $(\mathbf{x}_1, y_1), ..., (\mathbf{x}_n, y_n)$ be a training data set, where $\mathbf{x}_i \in \mathbb{R}^d, y_i \in \{-1, +1\}, i = 1, ..., n$.

Problem: By varying $\mathbf{w}, b$, maximize γ such that

$$y_i(\mathbf{w}\mathbf{x}_i + b) \geq \gamma \quad \text{for all } i = 1, ..., n \tag{1}$$

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- Replacing $\mathbf{w}$ and b by $2\mathbf{w}$ and $2b$ yields $y_i(2\mathbf{w}\mathbf{x}_i + 2b) \geq 2\gamma$
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Try again!

Problem Formulation: Solution

- Data set $(\mathbf{x}_i, y_i), i = 1, ..., n$ as before; let $H := \{\mathbf{x} \mid \mathbf{wx} + b = 0\}$ be the hyperplane given by $\mathbf{w}$ and b.
- Let
$$d(\mathbf{x}_i, H) := \min_x \{d(\mathbf{x}_i, \mathbf{x}) \mid \mathbf{wx} + b = 0\} \tag{2}$$
be the distance between $\mathbf{x}_i$ and H.
- *Solution:* Impose additional constraint: consider only combinations $\mathbf{w} \in \mathbb{R}^d, b \in \mathbb{R}$ such that for support vectors $\mathbf{x}$
$$y_i(\mathbf{wx} + b) \in \{-1, +1\} \tag{3}$$
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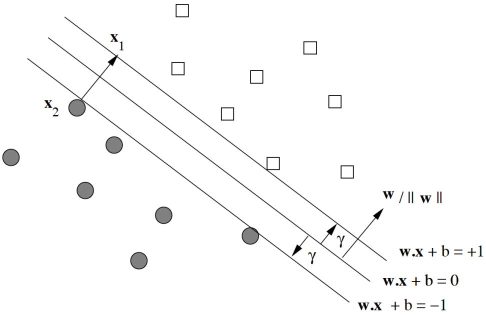

- $\mathbf{w}, b, \gamma$ determined according to (3),(4)
- $\mathbf{x}_2$ is support vector on lower hyperplane, so by (3), $\mathbf{w}\mathbf{x}_2 + b = -1$
- Let $\mathbf{x}_1$ be the projection of $\mathbf{x}_2$ onto upper hyperplane:

$$\mathbf{x}_1 = \mathbf{x}_2 + 2\gamma \frac{\mathbf{w}}{||\mathbf{w}||} \tag{5}$$

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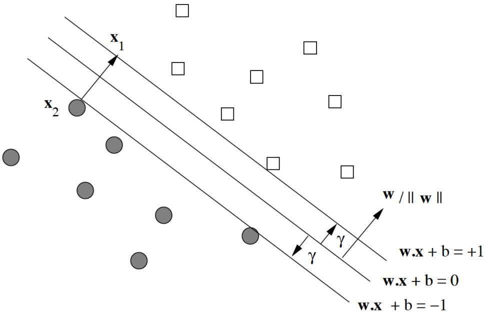

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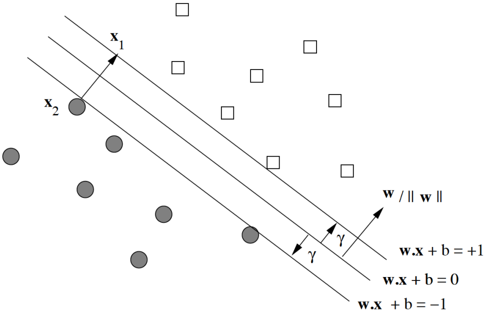

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Because $\mathbf{ww} = ||\mathbf{w}||^2$, and by further regrouping, we conclude that

$$\gamma = \frac{1}{||\mathbf{w}||} \tag{9}$$

Alternative Problem Formulation III

Let dataset $(\mathbf{x}_i, y_i), i = 1, ..., n$ be as before.

Equivalent Problem Formulation:

By varying $\mathbf{w}, b$, minimize $||\mathbf{w}||$ subject to

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Alternative Problem Formulation III

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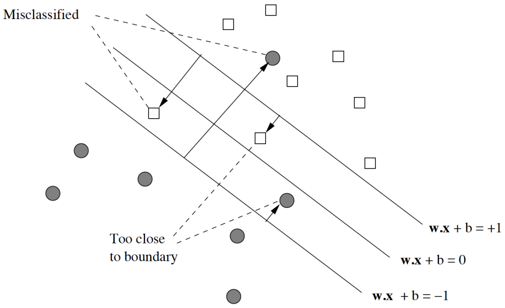
Optimizing under Constraints

- Topic is broadly covered
- Many packages can be used
- Target function $(||\mathbf{w}||)^2 = \sum_i w_i^2$ quadratic; well manageable

Example

Non Separable Data

NON SEPARABLE DATA SETS



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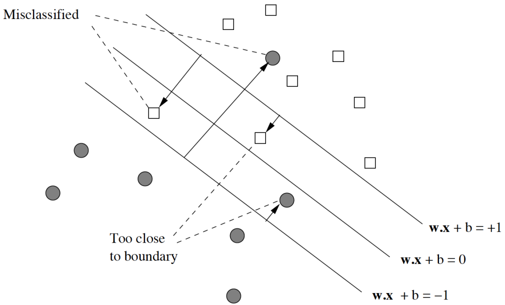

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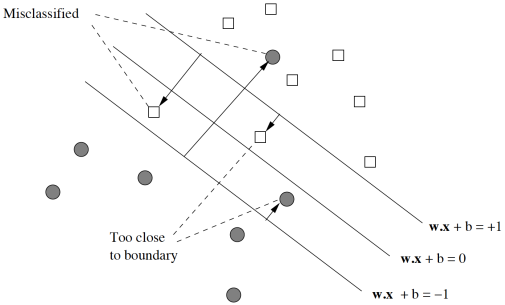

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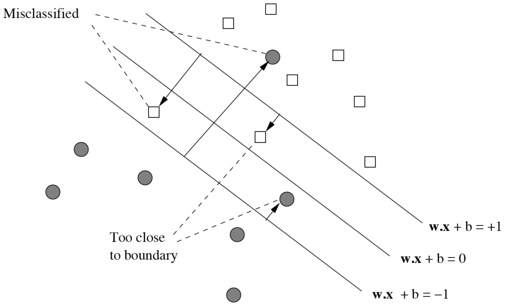

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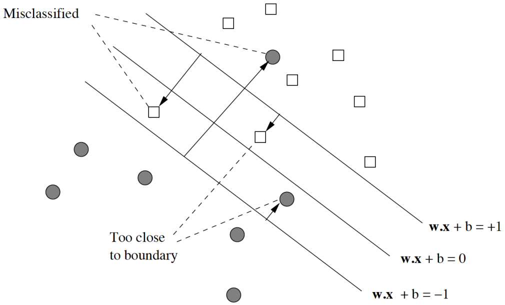

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and let $\mathbf{w} = (w_1, ..., w_d)$.

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Minimize the following function:

$$f(\mathbf{w}, b) = \frac{1}{2}\sum_{j=1}^{d} w_j^2 + C\sum_{i=1}^{n} \max\{0, 1 - y_i(\sum_{j=1}^{d} w_j x_{ij} + b)\} \qquad (11)$$

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- Minimizing $||\mathbf{w}||$ equivalent to minimizing monotone function of $||\mathbf{w}||$
  ☞ Minimizing f seeks minimal $||\mathbf{w}||$
- Vectors $\mathbf{w}$ and training data balanced in terms of basic units:

$$\frac{\partial(||\mathbf{w}||^2/2)}{\partial w_i} = w_i \quad \text{and} \quad \frac{\partial(\sum_{j=1}^{d} w_j x_{ij} + b)}{\partial w_i} = x_{ij}$$

- C is a regularization parameter
  - Large C: minimize misclassified points, but accept narrow margin
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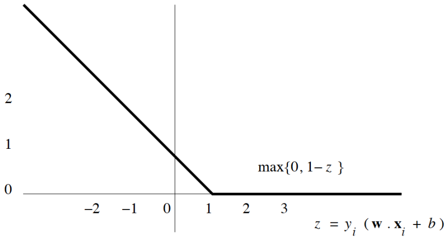
Non Separable Data: Hinge Function

Let the *hinge function* L be defined by

$$L(\mathbf{x}_i, y_i) = \max\{0, 1 - y_i(\sum_{j=1}^{d} w_j x_{ij} + b)\} \tag{12}$$



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Non Separable Data: Hinge Function

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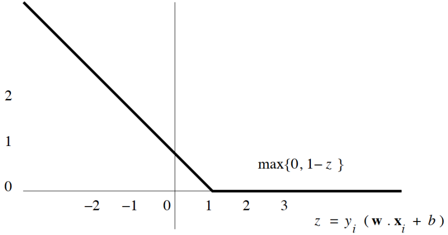

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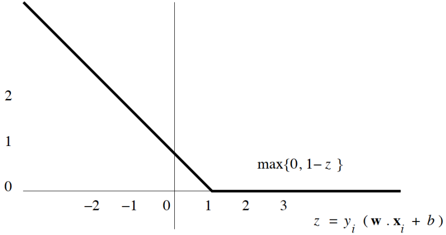

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$$\frac{\partial L}{\partial w_j} = \begin{cases} 0 & \text{if } y_i(\sum_{j=1}^{d} w_j x_{ij} + b) \geq 1 \\ -y_i x_{ij} & \text{otherwise} \end{cases} \tag{13}$$

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General / Further Reading

Literature

- Mining Massive Datasets , Chapter 12, Section 3: `http://infolab.stanford.edu/~ullman/mmds/ch12.pdf`

Thank you for listening!